CHAPTER TEN

CONDUCTING SCIENTIFIC INQUIRY

Conducting scientific inquiry accurately is an acquired art form. While no "scientific method" per se exists, scientists frequently conduct different types of experiments among which there is a certain amount of commonality. This chapter takes a broad rather than detailed approach to conducting a scientific experiment. There is no doubt that only a full course in the methods of scientific inquiry could even begin to scratch the surface of all the details required to conduct a scientific experiment in the most professional manner. This chapter should, however, provide the physics teacher candidate or high school physics teacher with sufficient information needed to conduct this form of inquiry at an age-appropriate level.

Types of Scientific Experiments

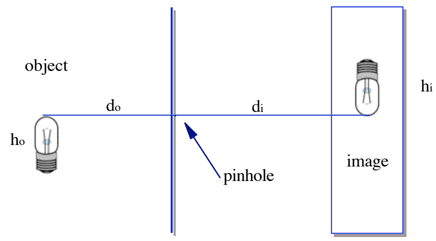
Observation
Testing
Application

Generic Experimental Design

While scientists will agree that there is no "scientific method" per se, there is basic agreement about what fundamental procedures can be followed for designing and conducting a physics experiment. For instance, consider the following steps:

1. Identify the problem to be solved.
2. Identify the system being studied.
3. Identify and distinguish system variables.
4. Identify the general procedure to be followed.
5. Identify the model if possible.
6. Choose the range for the variables.
7. Collect and interpret data.
8. Consider the overall precision of the experiment.

These steps, if nothing more, make it clear that a number of important factors need to be considered even for generic experimental design. Let's examine each of these factors in turn using a generic example. Refer to the following diagram as appropriate.



Step 1. Identify the problem to be solved. A physicist wants to experimentally determine the relationships associated with image formation in a pinhole camera (see the image above). The physicist seeks to find all the relationships between the height of an object, h_o, the height of its image, h_i, the distance of the object, d_o, and the distance of the image, d_i, as measured from the pinhole.

Step 2. Identify the system being studied. Just how the physicists solves the problem will depend upon the system being studied. Clearly, this problem deals with pinhole projection, so the system will consist of a light bulb (the object), a pinhole in a screen, and a screen where the image will form.

Step 3.Identify and distinguish system variables. Four of the system variables have already been clearly identified: h_i, h_o, d_i, and d_o. Are these the only system variables? No. There are other things that might vary in this experiment such as the size of the pinhole, the shape of the pinhole, the number of pinholes, the materials out of which the system is made, the kind of light bulb used, the nature of the material in which the pinhole is made and so on. Some variables are pertinent (e.g., h_i, h_o, d_i, and d_o), some are extraneous (e.g., probably the materials of which the system is made, the kind of light bulb used, the nature of the material in which the pinhole is made). There are other variables that are pertinent and must be controlled (e.g., the shape, size, and number of the pinholes). Other variables will be allowed to change and will be identified as either dependent or independent variables. Controlled variables are often known as system parameters.

Step 4. Identify the general procedure to be followed. Scientists conduct controlled experiments. In controlled experiments there will be only one independent and one dependent variable at any one time. Other variables are held constant and are considered during that phase of experimentation to be state variables. The reason we have only one independent and one dependent variable at a time is so that we can determine the unique relationship between these two variables. If two or more variables are allowed to change independent, there is no way of telling how much effect each of the independent variables has on the dependent variable. Clearly, in our experiment we need to allow four variables to change during different phases of the experiment, but this must be done in such a way that there is only one independent and one dependent variable at a time. The other pertinent variables will be held constant. To begin the experimental study, the physicist decides to see what how varying d_i affects h_i. During this phase of the experiment d_o and h_o are held constant. During additional phases of the experiment there will be other combinations of h_i, h_o, d_i, and d_o always with one independent, one dependent and two controlled variables.

Step 5. Identify the model if possible. Some times a theoretical analysis of the system will point to expected outcomes. This analysis can help interpret the data. In our hypothetical experiment, the physicist relies upon knowledge of the straight-line propagation of light and geometry to predict the outcome of the experiment. The real problem then is to determine whether or not experimental evidence supports the theoretical model. Sometimes it is not possible to directly predict the relationship being studied using theoretical approach, and such pragmatic approaches as dimensional analysis may be used to at least get a general understanding of what to expect.

Step 6. Choose the range for the variables. This is one of the most important considerations for a number of reasons. First, collecting only a small range of data it might be impossible to distinguish between, say, a linear model, and inverse model, and a power function model. Even a parabolic or hyperbolic function appear linear when a small portion of the curve is considered. Generally speaking, it is best to have as wide a range of data as possible. Determine the extrema, and then consider determining an appropriate number of points between the extrema at which to collect experimental data. Second, data collection is fraught with experimental error, and this error must be minimized to the greatest extent possible. Consider an absolute error (see *Glossary of Terms*) of 1mm. A 1mm absolute error isn't much when one is considering the distance, say, between the floor and the ceiling in a room.

The amount of relative error (see *Glossary of Terms*) is small. A 1*mm* absolute error is huge, for instance, if you are trying the measure the size of a tiny ball bearing. To increase the accuracy of an experiment it is important to reduce the amount of relative error in the data to a minimum.

Step 7. Collect and interpret data. In introductory physics labs this step will generally take the form of graphical analysis (see *Relationships from Graphs*). Using the computer program Graphical Analysis, graph the independent variable against the dependent variable. Perform regression analysis (after linearizing the data if appropriate) and determine the form of the relationship. Pay careful attention to how well your fit matches your theoretical model. If your theory base is correct, then the only reason there should be a difference between the calculated model and the theoretical model is due to random error in the experiment. When you conduct your regression analysis, keep in mind that a 5_{th} order polynomial will accurately fit almost any data. Fortunately, nature is much simpler that that. So, for instance, look for a trigonometric fit rather than a 5_{th} order polynomial fit if you identify an unusual curve. Additionally, be certain that you create a physically reasonable regression model. That is, if the variable on the x-axis is zero and you expect that the variable on the y-axis must similarly be zero, then the regression line must past through the origin (e.g., conduct regression analysis using a proportion fit, $y = mx$, instead of a linear fit, $y = mx + b$). Convert the algebraic expression into one that is physically interpreted. Replace y by the variable plotted on the y-axis, and x by the variable plotted on the x-axis. Determine the value and units of the slope and the y-intercept. Give these a physical interpretation if possible.

Step 8. Consider the overall precision of the experiment. If your experimental results in no way match your theoretical model (if used), then there are two sources of possible error. Either your model is wrong, or you data collection procedures contain a systematic error (see *Glossary of Terms*). You might be asked to use methods dealing with the propagation of error to determine the amount of expected error in a calculated term on the basis of the errors in the experimental terms.

Making Measurements

Errors, Accuracy, and Precision

In making physical measurements, one needs to keep in mind that measurements are not completely accurate. Each measurement will have some number of significant figures and should also have some indication as to how much we can "trust" it (i.e. error bars). Thus in order to reliably interpret experimental data, we need to have some idea as to the nature of the "errors" associated with the measurements. There are whole text books devoted to error analysis. We will not even attempt to be complete in our discussion of the topic, rather we will present some basic ideas to assist you in analyzing your data.

When dealing with error analysis, it is a good idea to know what we really mean by error. To begin with, let's talk about what error is not. Error is not a ***blunder*** such as forgetting to put the decimal point in right place, using the wrong units, transposing numbers, or any other silly mistake. Error is not your lab partner breaking your equipment. Error isn't even the difference between your own measurement and some generally accepted value. (That is a ***discrepancy***.) Accepted values also have errors associated with them; they are just better measurements than you will be able to make in a three-hour undergraduate physics lab. What we really mean by ***error*** has to do with uncertainty in measurements. Not everyone in lab will come up with the same measurements that you do and yet (with some obvious exceptions due to blunders) we may not give preference to one person's results over another's. Thus we need to classify types of errors.

Generally speaking, there are two types of errors: 1) ***systematic errors*** and 2) ***random errors***. Systematic errors are errors that are constant and always of the same sign and thus may not be reduced by averaging over a lot of data. Examples of systematic errors would be time measurements by a clock that runs too fast or slow, distance measurements by an inaccurately marked meter stick, current measurements by inaccurately calibrated ammeters, etc. Generally speaking, systematic errors are hard to identify with a single experiment. In cases where it is important, systematic errors may be isolated by performing experiments using different procedures and comparing results. If the procedures are truly different, the systematic errors should also be different and hopefully easily identified. An experiment that has very small systematic errors is said to have a high degree of ***accuracy***.

Random errors are a whole different bag. These errors are produced by any one of a number of unpredictable and unknown variations in the experiment. Examples might include fluctuations in room temperature, fluctuations in line voltage, mechanical vibrations, cosmic rays, etc. Experiments with very small random errors are said to have a high degree of ***precision***.
Since random errors produce variations both above and below some average value, we may generally quantify their significance using statistical techniques.

Dealing with Uncertainties

Uncertainty in Measurement

When scientists collect data, they rarely make just one measurement and leave it at that. Random error and unanticipated events can cause a single data point to be somewhat non-representative. Thus, multiple measurements are usually taken, and the "best value" is represented using a variety of approaches. Mean, median, and mode are measurement of central tendency. The degree to which data tend to be distributed around the mean value is known as variation or dispersion. A variety of measures of dispersion or variation around a mean are available, each with its own peculiar idiosyncrasy. The most common measures are the following: range, mean deviation, standard deviation, semi-interquartile range, and 10-90 percentile range. In this brief article we will examine only those measures most suitable for the purposes of an introductory lab experience.

As a scientist or an engineer you need to know how to deal effectively with uncertainty in measurement. Uncertainty in measurement is not to be confused with significant digits that have their own rules of uncertainty (see *Significant Figures in Measurement and Computation* elsewhere in this *Handbook*).

When one reads that some measurement has the value and absolute error of, say, d= 3.215m± .003m, how is this to be interpreted? The value 3.215m is known as the arithmetic mean (as opposed to weighted mean, geometric mean, harmonic mean, etc.) It is sometimes loosely called the average (but there are different types of averages as well). The arithmetic mean of N measurements is defined as follows:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Unless the person who provides the information indicates what the absolute error represents, there is usually no way of knowing. Does the indicated error represent the **range** of the values measured? That is, does this merely indicate that the "true" value of d is somewhere between 3.212m and 3.218m? It could imply this, but other interpretations are also possible.

In another case, ±0.003m could represent the **mean deviation.** The mean deviation is the mean of the absolute deviations of a set of data about the data's mean. The mean deviation is defined mathematically as

$$MD = \frac{1}{N}\sum_{i=1}^{N}\left|x_i - \bar{x}\right|$$

where $\bar{x}$ is the arithmetic mean of the distribution.

In yet another case ±0.003m could represent one **standard deviation**. This would then represent a probability that there is a 68% chance (±1σ or ±1standard deviation) that the "best" value is within the range of 3.212m and 3.218m. Such an interpretation is not strictly justified unless more than 30 data points have been used to calculate the mean value and standard deviation. In this interpretation data must also be "normally distributed" around the mean. That is, the mean, median, and mode of the data must be essentially the same. There must be a "bell shaped" distribution of data that are symmetrically distributed around the mean. Most data collected in introductory labs fail to meet these criteria.

Whatever form of error representation you use in lab, you must be consistent in your expressions of uncertainty in measurement. You must also declare what your error represents. For the purposes of these introductory labs, the following forms of uncertainty in measurement will be used if called for.

Example 1: Range – You use a meter stick with divisions marked off in 1*mm* units. The measurements of length appear to be somewhere between 93*mm* and 94*mm*. You would best represent the value of the length with uncertainty as follows stating without any ambiguity that the true value of the length measured lay somewhere between 93*mm* and 94*mm*.

$$93.5mm \pm 0.5mm$$

Example 2: Mean Deviation – You make six observations using an electronic photogate timer to measure nearly identical falls of a ball with the following results:

Observation 1: 6.556*s*
Observation 2: 6.576*s*
Observation 3: 6.630*s*
Observation 4: 6.587*s*
Observation 5: 6.575*s*
Observation 6: 6.602*s*

The value of the mean deviation is calculated thus:

$$\bar{x} = 6.588s$$
$$|x_1 - \bar{x}| = 0.022s$$
$$|x_2 - \bar{x}| = 0.012s$$
$$|x_3 - \bar{x}| = 0.042s$$
$$|x_4 - \bar{x}| = 0.001s$$
$$|x_5 - \bar{x}| = 0.013s$$
$$|x_6 - \bar{x}| = 0.014s$$

Sum of deviations = 0.104*s*
Mean deviation = 0.017*s*

The mean of the data along with mean deviation are accurately stated as 6.588*s* ± 0.017*s*

Example 3: Standard Deviation – Let's say you have made 30 measurements using your electronic timer. For the purposes of these introductory labs we will use the mean ±1standard deviation (or ±1σ) to represent the error. This process makes three major assumptions – that the variation in the data comes only from random error, that 30 or so data points are sufficient to calculate a good standard deviation, and that data constitute a "normal" (bell-shaped distribution) around the mean. None of these assumptions might be correct in a particular situation, so it might be a good idea to examine the distribution of data to make certain that data have the standard symmetrical "bell shaped" curve. Thus, interpretation of the standard deviation must be done with great caution.

Standard deviation of *N* data points can be readily calculated with the use of the following formula:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

Most scientific calculators will, given the data points, readily calculate the standard deviation with ease. Lab students should use this form to express uncertainty in repeated measures throughout these introductory lab activities when called for. Due caution should be observed in the use of significant digits.

Absolute and Relative Error

Absolute and relative error are two types of error with which every experimental scientist should be familiar. The differences are important.

Absolute Error: Absolute error is the amount of physical error in a measurement, period. Let's say a meter stick is used to measure a given distance. The error is rather hastily made, but it is good to $\pm 1mm$. This is the absolute error of the measurement. That is,

$$absolute\ error = \pm 1mm \text{ (0.001m).}$$

In terms common to *Error Propagation*

$$absolute\ error = \Delta x$$

where x is any variable.

Relative Error: Relative error gives an indication of how good a measurement is relative to the size of the thing being measured. Let's say that two students measure two objects with a meter stick. One student measures the height of a room and gets a value of 3.215 meters $\pm 1mm$ (0.001m). Another student measures the height of a small cylinder and measures 0.075 meters $\pm 1mm$ (0.001m). Clearly, the overall accuracy of the ceiling height is much better than that of the 7.5 cm cylinder. The comparative accuracy of these measurements can be determined by looking at their relative errors.

$$relative\ error = \frac{absolute\ error}{value\ of\ thing\ measured}$$

or in terms common to *Error Propagation*

$$relative\ error = \frac{\Delta x}{x}$$

where x is any variable. Now, in our example,

$$relative\ error_{ceiling\ height} = \frac{0.001m}{3.125m} \bullet 100 = 0.0003\%$$

$$relative\ error_{cylinder\ height} = \frac{0.001m}{0.075m} \bullet 100 = 0.01\%$$

Clearly, the relative error in the ceiling height is considerably smaller than the relative error in the cylinder height even though the amount of absolute error is the same in each case.

Propagation of Errors

Let's say that you have experimentally measured two quantities that will be used to determine a third quantity. There are errors in each of the measured quantities. How does the error in each of these two measured quantities affect the value of the calculated quantity?

Assume for the sake of the discussion that we use a pendulum to indirectly measure the acceleration due to gravity according to the hypothetical relationship

$$t = 2\pi\sqrt{\frac{l}{g}}$$

The length of the pendulum, l, and the period of oscillation, t, are measured. In the process of determining the local value of g, there is an error in the measurement of the length of the pendulum Δl. Similarly, assume an error in the measured period equal to Δt. How do these errors in length and period affect the resulting value of g? Consider the following process of analyzing error propagation. Given the above relationship that has reformulated to eliminate the square root and terms in the denominator, start by substituting $g + \Delta g$ for g, $t + \Delta t$ for t, and $l + \Delta l$ for l.

$$gt^2 = 4\pi^2 l$$
$$(g+\Delta g)(t+\Delta t)^2 = 4\pi^2(l+\Delta l)$$
$$(g+\Delta g)(t^2+2t\Delta t+\Delta t^2) = 4\pi^2(l+\Delta l)$$
$$gt^2+2gt\Delta t+g\Delta t^2+\Delta g t^2+2t\Delta g\Delta t+\Delta g\Delta t^2 = 4\pi^2 l+4\pi^2\Delta l$$

Eliminating terms in double and triple Δ (assumed << 1) and canceling like terms (note also that $gt^2 = 4\pi^2 l$) results in

$$\Delta g = \frac{4\pi^2\Delta l - 2gt\Delta t}{t^2}$$

Now, because the Δ terms can be considered either +/-, the maximum possible error in g will result when both error terms in the numerator are of like sign. That is, the maximum absolute error is given by the relationship

$$\Delta g_{\max} = \frac{4\pi^2\Delta l + 2gt\Delta t}{t^2}$$

After rearranging our terms and making a substitution based on our original identity ($gt^2 = 4\pi^2 l$) we get the relative error

$$\left(\frac{\Delta g}{g}\right)_{\max} = \frac{\Delta l}{l} + \frac{2\Delta t}{t}$$

Additional pointers for making this process work well:

1) Most of the time it is best to simplify a relationship before making the error term substitutions. Eliminating square roots and terms in the denominator will make the task of finding the error term much simpler. For instance, consider the following range equation with error terms in s, v, h, and g. Convert the equation before beginning the required substitutions:

$$s = v \times \sqrt{\frac{2h}{g}} \quad \text{becomes} \quad s^2 g = 2v^2 h \quad \text{then}$$

$$(s+\Delta s)^2(g+\Delta g) = 2(v+\Delta v)^2(h+\Delta h)$$

2) Pay close attention to substitutions that can be made by manipulating the simplified original equation obtained from following the previous hint (e.g. $2s = gt^2$). The reason for these substitutions is to eliminate "like" terms from both sides of the equation. This allows one to find terms that commonly appear in error propagation equations. In the example immediately above, this would mean the following:

$$\frac{\Delta v}{v}, \frac{\Delta s}{s}, \frac{\Delta g}{g}, \text{ and } \frac{\Delta h}{h}$$

It should be noted that with knowledge of partial differential equations, equations for absolute and relative error can be much more easily derived (especially with complex equations).

Percent Difference – Percent Error

Sometimes scientists will want to compare their results with those of others, or with a theoretically derived prediction. Each of these types of comparisons call for a different type of analysis, percent difference and percent error respectively.

Percent Difference: Applied when comparing two experimental quantities, E_1 and E_2, neither of which can be considered the "correct" value. The percent difference is the absolute value of the difference over the mean times 100.

$$\%\ Difference = \frac{|E_1 - E_2|}{\frac{1}{2}(E_1 + E_2)} \bullet 100$$

Percent Error: Applied when comparing an experimental quantity, E, with a theoretical quantity, T, which is considered the "correct" value. The percent error is the absolute value of the difference divided by the "correct" value times 100.

$$\%\ Error = \left|\frac{T - E}{T}\right| \bullet 100$$

Significant Figures

In making physical measurements, one needs to keep in mind that measurements are not completely accurate. Each measurement will have some number of significant figures and should also have some indication as to how much we can "trust" it (i.e. error bars). Thus in order to reliably interpret experimental data, we need to have some idea as to the nature of the "errors" associated with the measurements. There are whole text books devoted to error analysis. We will not even attempt to be complete in our discussion of the topic, rather we will present some basic ideas to assist you in analyzing your data.

<u>Significant Figures</u>

The significant figures in a quantity are the meaningful digits in it. There are several conventions listed below to keep in mind when dealing with significant figures:

1) Any nonzero digit is a significant digit. Examples: 3.14 has three significant digits and 239.67 has five significant digits.
2) Zeros in between nonzero values are significant digits. Example 20.4 has three significant digits.
3) Zeros to the left of the first nonzero digit are NOT significant. Example: 0.0012 has two significant digits.
4) For numbers with decimal points, zeros to the right of nonzero digits are significant. Example: Both 2.0 and 0.0020 have two significant digits.
5) For numbers without decimal points, zeros to the right of nonzero digits may or may not be significant. Example: 350 may have two or three significant digits. To avoid confusion, use a decimal point: 350. has three significant digits, 350 has two.

6) The last significant digit in a number should have the same order of magnitude as the uncertainty. Examples: 3.1415±0.0001; 0.8±0.2
7) The uncertainty should be rounded to one or two significant digits.
8) When "doing" mathematical manipulations with numbers that have varying numbers of significant digits, the final result cannot have more significant digits than the least well known number. In practice, we will keep all the numbers to their known accuracy through out the calculation and then round to the number of significant figures at the end. Examples: 2.0*3.14159 = 6.3; 25.001+32.0 = 55.0

Conversion Factors

The use of conversion factors will not be confusing so long as certain rules are adhered to rigidly. These rules can best be shown with the use of examples:

Problem: Convert 3 (assumed an integer) inches (*in*) to millimeters (*mm*).

The conversion factor is $1in = 25.4mm$ precisely. This can be rewritten in two ways:

$$\frac{1in}{25.4mm} = 1 \qquad \text{or} \qquad \frac{25.4mm}{1in} = 1$$

Now, we can multiply any quantity by 1 without changing its *essential* value. We can multiply 3 inches by 1 and still have 3 inches. If we choose to rewrite 1 in one of the above forms, we can convert a distance in one system of measure (British) to that of another (metric). The numerical value will not be the same because the unit of measurement will change. However, the actual distance will be the same in either system of measurement. The question is, which one of the above two ways should we choose to write 1 so as to successfully convert from one system of measurement to another?

If we choose to write 1 in the first way, we have:

$$3in = 3in \times 1 = 3in \times \frac{1in}{25.4mm} = \frac{0.118in^2}{mm}$$

The result is in mixed units that have no direct physical meaning and is a combination of both metric and British systems. If we choose to write 1 in the second way, we have:

$$3in = 3in \times 1 = 3in \times \frac{25.4mm}{1in} = 76.2mm$$

Note that the inches unit has canceled; the result is purely metric.

What happens if one needs to convert, say, cubic centimeters (cm^3) into cubic meters (m^3)? Consider the following problem.

Problem: Convert 300cm^3 into cubic meters.

The conversion factor is $100cm$ = 1 meter. Now, one cannot merely multiply $300cm^3$ by $1m/100cm$ to arrive at the answer. If this were done, the answer would come out in cm^2 x m and, again, mixed units. To handle this sort of complication, we must rewrite 1 as 1^3. One cubed is the same as one. For example:

$$300cm^3 = 300cm^3 \times 1^3 = 300cm^3 \times (\frac{1m}{100cm})^3$$

$$= 300cm^3 \times \frac{1m^3}{1\times10^6 cm^3} = \frac{300m^3}{10^6} = 3\times10^{-4} m^3$$

When using conversion factors, always pay meticulous attention to the units and their associated powers. In doing so, you need never make an error in converting from one system to another.

Dimensional Analysis

Scientists sometimes will use a process called dimensional analysis to predict the form of the relationship between variables in a system. While this does NOT constitute a theory base, it does give scientists a better understanding of what to expect experimentally, and helps them to design scientific experiments. Carefully observe the following process of dimensional analysis for a cart accelerating down a fixed inclined plane.

Let's say that a scientist observes the motion of the cart on the fixed incline, and after conducting a few qualitative experiments concludes that the distance of the cart is a function of its acceleration and the time. Assume that the cart accelerates from rest when the stopwatch starts. What is the expected form of the relationship between distance, acceleration, and time?

The form of this relationship can be predicted using an approach known as dimensional analysis. This process is based on the knowledge that the distance (d) is a function of the acceleration (a) and the time (t). That is,

$$d = f(a,t)$$

Now, if d (expressed in meters, m) is related to both a (expressed in meters per second squared, m/s^2) and t (expressed in seconds, s) in some form, then the only thing missing is a proportionality constant and the powers of the variable terms. Write a proportionality applying power terms x and y to a and t respectively. Replace variables by units and solve for the power terms as, in this case, two simultaneous equations with two unknowns. Working backward, then find the form of the equation and insert proportionality constant. (Note that if the unit of time, s, is present on the right side of the equation, it must also be present on the left side of the equation. Place s^0 on the left side of the equation in step two below as shown; s^0 is equal to 1 and does not affect the equality.)

$d \propto a^x t^y$

replacing variables with units

$m^1 s^0 \propto \left(\frac{m}{s^2}\right)^x s^y$

simplifying

$m^1 s^0 \propto m^x s^{y-2x}$

equating exponents on m and s

$1 = x$

$0 = y - 2x$

and solving simultaneous equations

$y = 2x$

hence, $y = 2$

thus, $d \propto a^1 t^2$

or, $d = kat^2$

Experiments can be conducted or theoretical work performed to find the value of the constant, k. In reality, k equals ½. This gives the familiar kinematic equation

$$d = \frac{1}{2}at^2$$

Practices in the Laboratory

Evaluating an Experiment

Chi-Square Test for Goodness of Fit
(after *Applied Statistics* by Hinkle/Wiersma/Jurs)

Scientists will often use the Chi-square (χ^2) test to determine the goodness of fit between theoretical and experimental data. In this test, we compare **observed values** with **theoretical or expected values.** Observed values are those that the researcher obtains empirically through direct observation; theoretical or expected values are developed on the basis of some hypothesis. For example, in 200 flips of a coin, one would expect 100 heads and 100 tails. But what if 92 heads and 108 tails are observed? Would we reject the hypothesis that the coin is fair? Or would we attribute the difference between observed and expected frequencies to random fluctuation?

Consider another example. Suppose we hypothesize that we have an unbiased six-sided die. To test this hypothesis, we roll the die 300 times and observe the frequency of occurrence of each of the faces. Because we hypothesized that the die is unbiased, we expect that the number on each face will occur 50 times. However, suppose we observe frequencies of occurrence as follows:

Face value	*Occurrence*
1	42
2	55
3	38
4	57
5	64
6	44

Again, what would we conclude? Is the die biased, or do we attribute the difference to random fluctuation?

Consider a third example. The president of a major university hypothesizes that at least 90 percent of the teaching and research faculty will favor a new university policy on consulting with private and public agencies within the state. Thus, for a random sample of 200 faculty members, the president would *expect* 0.90 x 200 = 180 to favor the new policy and 0.10 x 200 = 20 to oppose it. Suppose, however, for this sample, 168 faculty members favor the new policy and 32 oppose it. Is the difference between observed and expected frequencies sufficient to reject the president's hypothesis that 90 percent would favor the policy? Or would the differences be attributed to chance fluctuation?

Lastly, consider an experimental result where, for given independent values of "X," the following theoretical (expected) and experimental (observed) dependent values of Y were found:

X	$Y_{theoretical}$	$Y_{experimental}$
3.45	11.90	11.37
4.12	16.97	17.02
4.73	22.37	23.78

5.23	27.35	26.13
6.01	36.12	35.96
6.82	46.51	45.22
7.26	52.71	53.10

In each of these examples, the test statistic for comparing observed and expected frequencies is χ^2, defined as follows:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O-E)^2}{E}$$

where

O = observed value
E = expected value
k = number of categories, groupings, or possible outcomes

The calculations of χ^2 for each of the three examples, using the above formula, are found in Tables 1-4.

TABLE 1. Calculation of χ^2 for the Coin-Toss Example

Face	**O**	**E**	**O-E**	**(O-E)^2**	**(O-E)^2/E**
Heads	92	100	-8	64	.64
Tails	108	100	+8	64	.64
Totals	200	200	0	--	1.28 = χ^2

TABLE 2. Calculation of χ^2 for the Die Example

Face Value	**O**	**E**	**O-E**	**(O-E)^2**	**(O-E)^2/E**
1	42	50	-8	64	1.28
2	55	50	5	25	.50
3	38	50	-12	144	2.88
4	57	50	7	49	.98
5	64	50	14	196	3.92
6	44	50	-6	36	.72
Totals	300	300	0	--	10.28= χ^2

TABLE 3. Calculation of χ^2 for the Consulting-Policy Example

Disposition	**O**	**E**	**O-E**	**(O-E)^2**	**(O-E)^2/E**
Favor	168	180	-12	144	.80
Oppose	32	20	+12	144	7.20
Totals	200	200	0	--	8.00= χ^2

TABLE 4. Calculation of χ^2 for the Experiment Example

X	**O**	**E**	**O-E**	**(O-E)^2**	**(O-E)^2/E**
3.45	11.37	11.90	-.53	.28	.02
4.12	17.02	16.97	.05	.02	.00
4.73	23.78	22.37	1.41	1.99	.09
5.23	26.13	27.35	-1.22	1.49	.05
6.01	35.96	36.12	-.16	.03	.00
6.82	45.22	46.51	-1.29	1.66	.04
7.26	53.10	52.71	.39	.15	.00
Totals	212.58	213.93	-1.35	5.62	0.20 = χ^2

There is a family of χ^2 distributions, each a function of the degrees of freedom associated with the number of categories in the sample data. Only a single degree of freedom (df) value is required to identify the specific χ^2distribution. Notice that all values of $\chi^2$are positive, ranging from zero to infinity.

Consider the degrees of freedom for each of the above examples. In the coin example, note that the *expected* frequencies in each of the two categories (heads or tails) are *not* independent. To obtain the expected frequency of tails (100), we need only to subtract the expected frequency of heads (100) from the total frequency (200), or 200 - 100 = 100. Similarly, for the example of the new consulting policy, the expected number of faculty members who oppose it (20) can be found by subtracting the expected number who support it (180) from the total number in the sample (200), or 200 - 180 = 20. Thus, given the expected frequency in one of the categories, the expected frequency in the other is readily determined. In other words, only the expected frequency in one of the two categories is free to vary; that is, there is only 1 degree of freedom associated with these examples.

For the die example, there are six possible categories of outcomes: the occurrence of the six faces. Under the assumption that the die is fair, we would *expect* that the frequency of occurrence of each of the six faces of the die would be 50. Note again that the expected frequencies in each of these categories are *not* independent. Once the expected frequency for five of the categories is known, the expected frequency of the sixth category is uniquely determined, since the total frequency equals 300. Thus, only the expected frequencies in five of the six categories are free to vary; there are only 5 degrees of freedom associated with this example.

In the last example dealing with the data from the experiment, the cross-tabulation table has 2 columns and 7 rows. The degrees of freedom for such tables is (# rows – 1)(# columns – 1) = (6)(1) = 6

The Critical Values for the χ^2 Distribution

The use of the χ^2 distribution in hypothesis testing is analogous to the use of the *t* and F distributions. A null hypothesis is stated, a test statistic is computed, the observed value of the test statistic is compared to the critical value, and a decision is made whether or not to reject the null hypothesis. For the coin example, the null hypothesis is that the frequency of heads is equal to the frequency of tails. For the die example, the null hypothesis is that the frequency of occurrence of each of the six faces is the same. In general, it is not a requirement for the categories to have equal expected frequencies. For instance, in the example of the new consulting policy, the null hypothesis is that 90 percent of the faculty will support the new policy and 10 percent will not.

The critical values of χ^2 for 1 through 30 degrees of freedom are found in Table 5. Three different percentile points in each distribution are given – $\alpha = .10$, $\alpha = .05$, and $\alpha = .01$. (e.g., chances of 10%, 5%, and 1% respectively of rejecting the null hypothesis when it should be retained). For the coin and consulting-policy examples, the critical values of χ^2 for 1 degree of freedom, with $\alpha = .05$ and $\alpha = .01$, are 3.841 and 6.635, respectively. For the die example, the corresponding critical values of χ^2 for 5 degrees of freedom are 11.070 and 15.086. Although Table 5 is sufficient for many research settings in the sciences, there are some situations in which the degrees of freedom associated with a χ^2 test are greater than 30. These situations are not addressed here.

Now that we have seen the table of critical values for the χ^2 distribution, we can complete the examples. For the coin example, the null hypothesis is that the frequency of heads equals the frequency of tails. As we mentioned, because there are only two categories, once the expected value of the first category is determined, the second is uniquely determined. Thus, there is only 1 degree of freedom associated with this example. Assuming that the $\alpha = .05$ level of significance is used in testing this null hypothesis, the critical value of χ^2 (χ^2_{CV}) is 3.841 (see Table 5). Notice that, in Table 1, the calculated value of χ^2 is 1.28. Because the calculated value does *not* exceed the critical value, the null hypothesis (the coin is fair) is *not* rejected; the differences between observed and expected frequencies are attributable to chance fluctuation. That is, when the calculated value of χ^2 exceeds the critical value, the data support the belief that a significant difference exists between expected and actual values.

For the example of the new consulting policy, the null hypothesis is that 90 percent of the faculty would support it and 10 percent would not. Again, because there are only two categories, there is 1 degree of freedom associated with the test of this hypothesis. Thus, assuming $\alpha = .05$, the χ^2_{CV} is 3.841. From Table 3, we see that the calculated value of χ^2 is 8.00; therefore, the null hypothesis (that there is no difference) is rejected. The conclusion is that the percentage of faculty supporting the new consulting policy is not 90.

For the die example, the null hypothesis is that the frequency of occurrence of each of the six faces is the same. With six categories, there are 5 degrees of freedom associated with the test of this hypothesis; the χ^2_{CV} for $\alpha = .05$ is 11.070. Using the data from Table 2, $\chi^2 = 10.28$. Because this calculated value is less than the critical value, the null hypothesis is retained. The conclusion is that the differences between the observed and expected frequencies in each of the six categories are attributable to chance fluctuation.

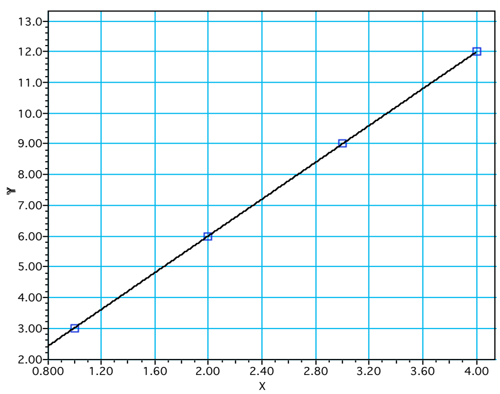
For the example with the experiment, the null hypothesis is that there is no difference between theoretical and experimental results. With seven data pairs there are 6 degrees of freedom associated with the test of this hypothesis; the χ^2_{CV} for $\alpha = .05$ is 12.592. Because this calculated value of χ^2 (0.20) is less than the critical value (12.592), the null hypothesis is retained. The conclusion is that the differences between the observed and expected values in each of the seven data pairs are attributable to chance fluctuation. The experimental results are therefore consistent with the theoretical results to with a 95% chance of probability.

TABLE 5. CRITICAL χ^2 VALUES FOR UP TO 30 DEGREES OF FREEDOM

df	$\alpha = .10$	$\alpha = .05$	$\alpha = .01$
1	2.706	3.841	6.635
2	4.605	5.991	9.210
3	6.251	7.815	11.345
4	7.779	9.488	13.277
5	9.236	11.070	15.086
6	10.645	12.592	16.812
7	12.017	14.067	18.475
8	13.362	15.507	20.090
9	14.684	16.919	21.666
10	15.987	18.307	23.209
11	17.275	19.675	24.725
12	18.549	21.026	26.217
13	19.812	22.362	27.688
14	21.064	23.685	29.141
15	22.307	24.996	30.578
16	23.542	26.296	32.000
17	24.769	27.587	33.409
18	25.989	28.869	34.805
19	27.204	30.144	36.191
20	28.412	31.410	37.566
21	29.615	32.671	38.932
22	30.813	33.924	40.289
23	32.007	35.172	41.638
24	33.196	36.415	42.980
25	34.382	37.652	44.314
26	35.563	38.885	45.642
27	36.741	40.113	46.963
28	37.916	41.337	43.278
29	39.087	42.557	49.558
30	40.256	43.773	50.892

Common Graph Forms in Physics

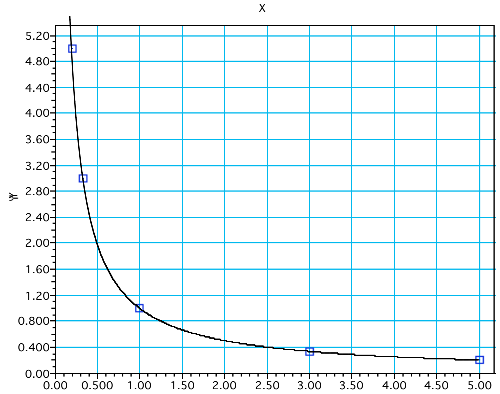
Working with graphs – interpreting, creating, and employing – is an essential skill in the sciences, and especially in physics where relationships need to be derived. As an introductory physics student you should be familiar with the typical forms of graphs that appear in physics. Below are a number of typical physical relationships exhibited graphically using standard X-Y coordinates (e.g., no logarithmic, power, trigonometric, or inverse plots, etc.). Study the forms of the graphs carefully, and be prepared to use the program *Graphical Analysis* to formulate relationships between variables by using appropriate curve-fitting strategies. Note that all non-linear forms of graphs can be made to appear linear by "linearizing" the data. Linearization consists of such things as plotting X versus Y^2 or X versus 1/Y or Y versus log (X), etc. Note: While a 5th order polynomial might give you a better fit to the data, it might not represent the simplest model.



LINEAR RELATIONSHIP: What happens if you get a graph of data that looks like this? How does one relate the X variable to the Y variable? It's simple, Y = A + BX where B is the slope of the line and A is the Y-intercept. This is characteristic of Newton's second law of motion and of Charles' law:

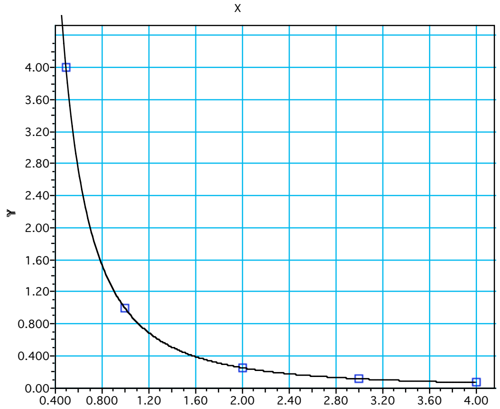
$$F = ma$$

$$\frac{P}{T} = const.$$



INVERSE RELATIONSHIP: This might be a graph of the pressure and temperature for a changing volume constant temperature gas. How would you find this relationship short of using a computer package? The answer is to simplify the plot by manipulating the data. Plot the Y variable versus the inverse of the X variable. The graph becomes a straight line. The resulting formula will be Y = A/X or XY = A. This is typical of Boyle's law:

$$PV = const.$$

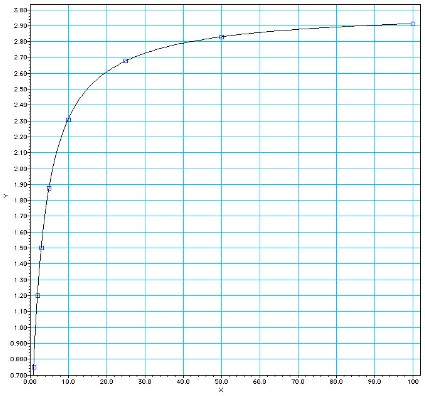


INSERSE-SQUARE RELATIONSHIP: Of the form Y = A/X^2. Characteristic of Newton's law of universal gravitation, and the electrostatic force law:

$$F = \frac{Gm_1m_2}{r^2}$$

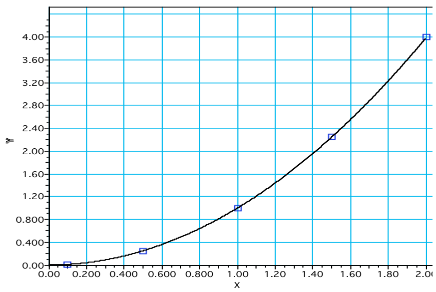
$$F = \frac{kq_1q_2}{r^2}$$

In the latter two examples above there are only subtle differences in form. Many common graph forms in physics appear quite similar. Only be looking at the "RMSE" (root mean square error provided in *Graphical Analysis*) can one conclude whether one fit is better than another. The better fit is the one with the smaller RMSE. See page 2 for more examples of common graph forms in physics.



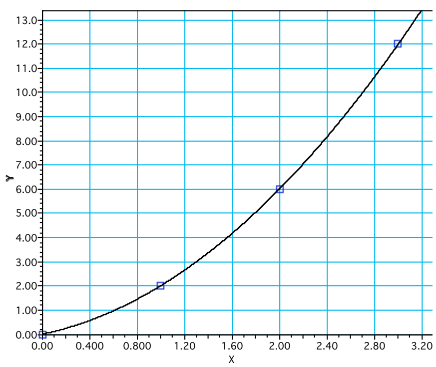
DOUBLE-INVERSE RELATIONSHIP: Of the form 1/Y = 1/X + 1/A. <u>Most readily identified by the presence of an asymptotic boundary (y = A) within the graph.</u> This form is characteristic of the thin lens and parallel resistance formulas.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \quad \text{and} \quad \frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$$



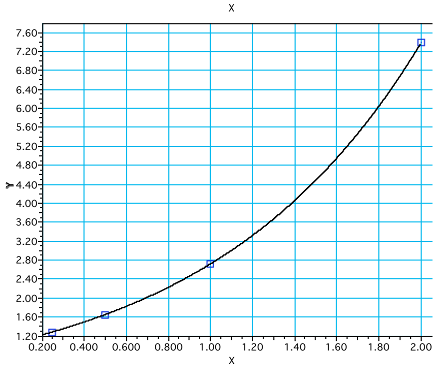
POWER RELATIONSHIP: Of the form Y = AX^B. Typical of the distance-time relationship:

$$d = \frac{1}{2}at^2$$



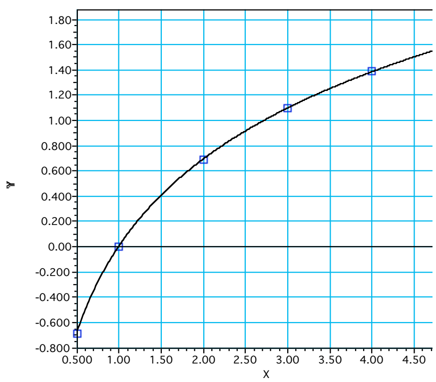
POLYNOMIAL OF THE SECOND DEGREE: Of the form Y = AX + BX^2. Typical of the kinematics equation:

$$d = v_o t + \frac{1}{2}at^2$$



EXPONENTIAL RELATIONSHIP: Of the form Y = A * exp(BX). Characteristic of exponential growth or decay. Graph to left is exponential growth. The graph of exponential decay would look not unlike that of the inverse relationship. Characteristic of radioactive decay.

$$N = N_o e^{-\lambda t}$$



NATURAL LOG (LN) RELATIONSHIP: Of the form Y = A ln(BX). Characteristic of entropy change during a free expansion:

$$S_f - S_i = nR\ln\frac{V_f}{V_i}$$

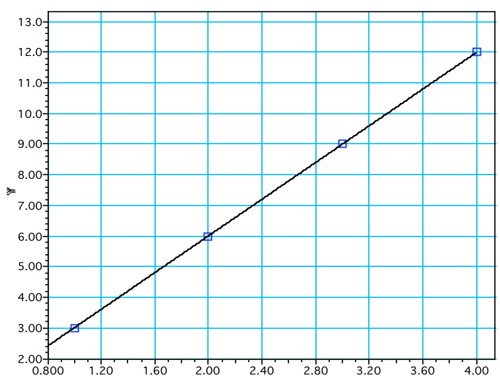
Graphing in Physics

(*Version 2*)

Comments by Carl J. Wenning

Working with graphs -- interpreting, creating, and employing -- is an essential skill in the sciences, and especially in physics where relationships need to be derived. As a physics student you probably have used graphing hundreds of times and are pretty much familiar with both the interpretation and creation of graphs. Nonetheless, my experience in PHY 302 is that most students are unaware of how powerful is the use graphs to determine relationships between variables.

Below are a number of typical physical relationships exhibited graphically. Study the forms of the graphs carefully and be prepared to help your students formulate relationships between variables.



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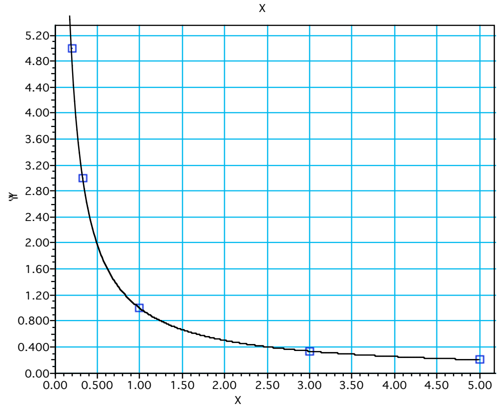

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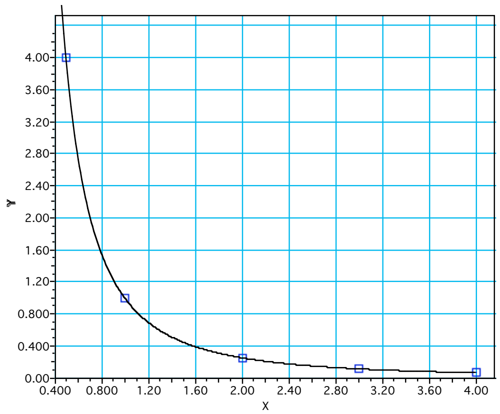

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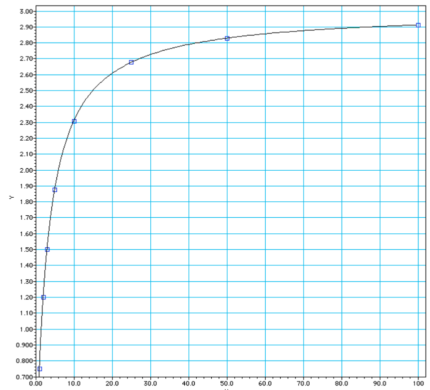

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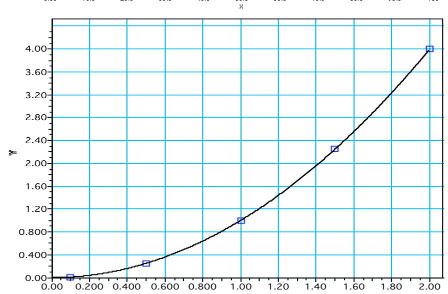

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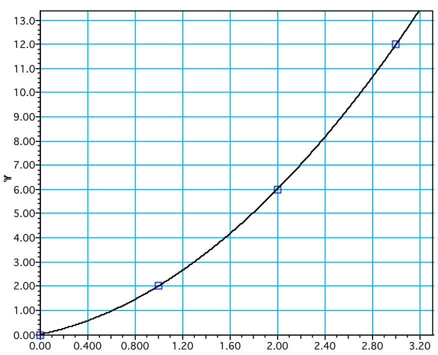

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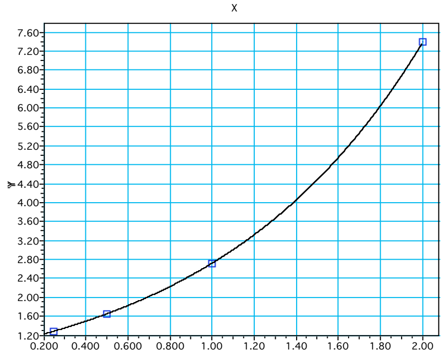

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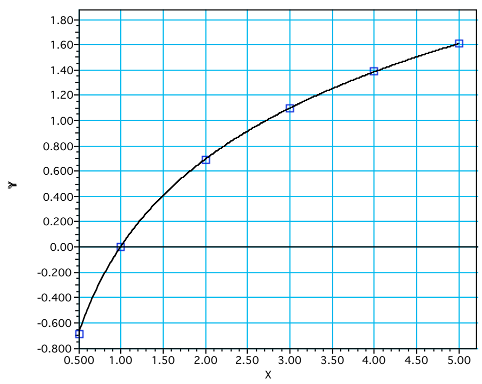

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EXAMPLES OF COMMON GRAPH CONSTRUCTION ERRORS

Data collection errors. If you want straight-line data, collect only data for only two points; if reproducibility is a problem, conduct only one test. A straight line can always be made to fit through two points. Clearly this is a wrong way to collect and interpret data for, indeed, a line and, indeed, any curve can be made to pass through just two points. Observations should be repeated for the sake of accuracy.

Confusing independent and dependent variables. The ***I***ndependent variable is what ***I*** change." The size of the ***D***ependent variable ***D***epends on the value of the independent variable.

Putting wrong variables on axes. For a matter of convenience, it is common practice to put the independent variable on the horizontal x-axis (bottom) rather than the vertical y-axis (side) when seeking a relationship to define the dependent variable. For instance, if one wants to arrive at a relationship describing ***F*** as a function of ***a***, then ***F*** (the dependent variable) should be plotted on the y-axis. The slope becomes the proportionality constant, ***F = ma***.

Failure to understand the significance of "linearizing" data. When data are non-linear (not in a straight line when graphed), it is best to "linearize" the data. This does not mean to fit the curved data points with a straight line. Rather, it means to modify one of the variables in some manner such that when the data are graphed using this new data set, the resulting data points will appear to lie in a straight line. For instance, say the data appear to be an inverse function – as x is doubled, y is halved. To linearize the data for such a function plot x versus 1/y. If this is indeed an inverse function, then the plot of x versus 1/y date will be linear.

Failure to properly relate y = mx + b to the linearized data. When plotting, say, distance (on the y-axis) versus time (on the x-axis), the correct relationship between distance and time can be found by relating y to distance, x to time, m to the slope, and b to the y-intercept. For instance, data have been linearized for the function resulting in a straight-line graph when distance is plotted versus time-squared. The slope is $2 m/s^2$ and the y-intercept is 1 m. The correct form of the relationship between all the variables will be distance = $(2m/s^2)$*$time^2$ + 1 m.

Failure to apply the appropriate physical model to the data. When students have a scattering of data, they might at first be tempted to fit the data with a linear model, y = mx + b. Using such a linear fit will result in an error if when y equals 0, x must also be 0. For instance, if the force on a spring is 0, its extension from the equilibrium position will be zero. This is, no force, no extension. Such a relationship expressed graphically MUST pass through the origin. Because of errors in the collected data, a linear fit will almost never pass through the origin as it should. The way to solve this problem is to conduct a proportional fit, y = mx. This forces b = 0 and assures that the best fit line will pass through the origin.

Using multiple (0,0) points to force best-fit line through origin. Some times students justifiably realize that a best-fit line for a straight-line relationship should pass through the origin (e.g., if force equals zero, then acceleration must also equal zero). They then include multiple (0,0) points in the data set in an effort to force the best-fit line through the origin. This rarely works. Better is to fit the linearized data with a proportional relationship (y = mx) in these cases rather than a linear relationship (y = mx + b). The value of b (the y-intercept) is forced to be zero in the proportional fit.

Failure to apply appropriate labeling. Each graph should be appropriately labeled; each axis should be similarly labeled with its variable and units (in parentheses). For instance, time (seconds) or distance (meters).

Confusing variables with units. Don't confuse variables (time, resistance, distance) with their units (seconds, ohms, meters).

Connecting data points with straight lines. Never connect even linearized data point-to-point with straight lines. "Mother nature is not jerky." Linearized data sometimes appears not to fit the best-fit line precisely. This is frequently due to errors in the data.

Scaling errors. In physics it is often best to show the origin (0,0) in each graph. While this is not an error, it frequently leads to misinterpretations of relationships. Another problem to avoid is not sticking with a consistent scale on an axis. Values should be equally spaced on graphs. For instance, the distances between 1, 2, 3, and 4 should be the same. To be avoided are situations where the spacing between such numbers is irregular or inconsistent.

Beware the high order polynomial fit. Mother Nature tends to be quite simple, but any 4^{th} or 5^{th} order polynomial function can be made to fit nearly any given set of data. Avoid fitting data with non-linear functions of above second order (e.g., $y = a + bx + cx^2$). Simple functions like x = constant / y or x = A*sin(θ) are much more common.

EXAMPLES OF COMMON GRAPH INTERPRETATION ERRORS

Failing to note that when the slope is zero (horizontal line), there is no relationship. Some times students think that a relationship between two variables exists when, in fact, it does not. This often happens with auto-scaling computer graphing programs where, for instance, the vertical axis runs from 0.05 to 0.06 and horizontal axis reads from 0 to 10. The 0.05-0.06 range of the vertical axis is automatically stretched out to take up the same linear space as the horizontal axis thus "filling the page." In this case the actual relationship might be y = 0.0001x. Some falsely think that this constitutes a meaningful relationship. It is an extremely weak relationship probably arising from measurement errors. By rescale each axis, the graph clearly shows a line of essentially zero slope.

Failing to properly interpret horizontal and vertical slopes. When the slope of a line is zero degrees (horizontal), then there is no relationship between the plotted variables. When the slope of a line is 90 degrees (vertical), there is no relationship between the plotted variables. In both these cases, the so-called "dependent" variable does not actually depend upon the independent variable.

Failing to properly interpret a negative slope. A negative slope does NOT imply an inverse relationship. What it does imply is that when x gets larger, y becomes larger but in the negative direction. y = -mx is a negative slope. xy = m or x = m/y is an inverse relationship. Additionally, a negative acceleration does not necessary imply a slowing down of an object. An object moving in the –x direction will, in fact, be increasing in speed.

Failing to properly interpret the y-intercept. The value of the y-intercept represents the value of the dependent variable when the independent variable is zero. Sometimes when lines or curve representing relationships clearly should pass through the origin (0, 0), they do not. The value of the y-intercept is then small but not zero. This is usually the result of data collection errors. Linearized data can be "forced through the origin" by conducting a proportional fit of the data (y = mx) rather than a linear fit (y = mx + b).

Finding slope using two data points NOT on best-fit line. Finding the slope of the best-fit line from two data points rather than two points on the best-fit line is a common error. This can result in a substantial error in the slope. Use two points on the best-fit line for this process, not data points to find the slope of a line.

Improperly finding the slope using elements of a single coordinate point. When finding the slope of a tangent line, students sometimes will mistakenly take the coordinate point nearest where the tangent line is drawn and from those coordinates attempt to determine the slope. For instance, at (x, y) = (5s, 25m) the data point values are mistakenly used to find a slope of 25m/5s = 5m/s.

Failing to recognize trends. Graphs must be properly interpreted through the use of generalizations (e.g., double the mass, and the period increases by 4)

Improperly interpreting the physical meaning of the slope. The units of a slope are a give away as far as interpreting the physical meaning of a slope is concerned. The slope is defined as the change in the y value divided by a corresponding change in the x value. Hence, the units are those of the y-axis divided by those of the x-axis. If distance

(m) is on the y-axis, and time (s) on the x-axis, then the slope's units will be m/s which is the unit for speed. Similarly, if velocity (m/s) is on the y-axis and time (s) is on the x-axis, then (m/s)/s = m/s², the unit of acceleration. Slopes can be either positive or negative.

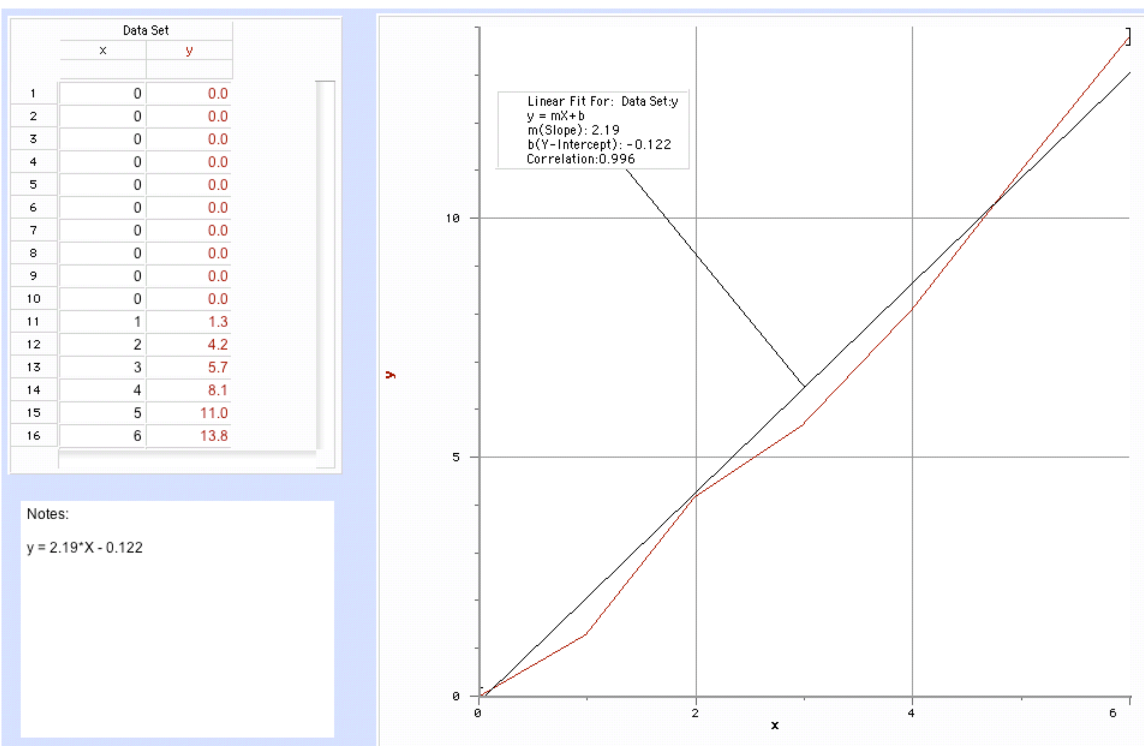
Misinterpreting the area under a best-fit line or curve. To more easily determine what the area under the curve represents, examine the product of the units on the x- and y-axes. For instance, if velocity or speed (m/s) is on the y-axis and time (s) is on the x-axis, the product of the units will be meters that is the unit of displacement or distance. Hence, the area under the curve of a v-t graph is displacement. Similarly, the area under the curve of an a-t graph is velocity.

Not recognizing errors in data. When one graphs data, large errors in measurement usually stand out. They become noticeable when they deviate from the trend of the data. Such data points should not only be questioned, but the data collection process for this datum repeated to see if there is a readily explainable error. Suspect data should be struck through rather than erased in the written record. Update the data file appropriately.

Not understanding the meaning of the best-fit line. A best-fit line is a line which, when drawn, minimizes the sum of the squares of the deviations from the line usually in the y direction. It is the line that most closely approximates the trend of the data.

Preparing Graphs: Requirements for Introductory Physics Lab Activities

How NOT to Prepare a Graph



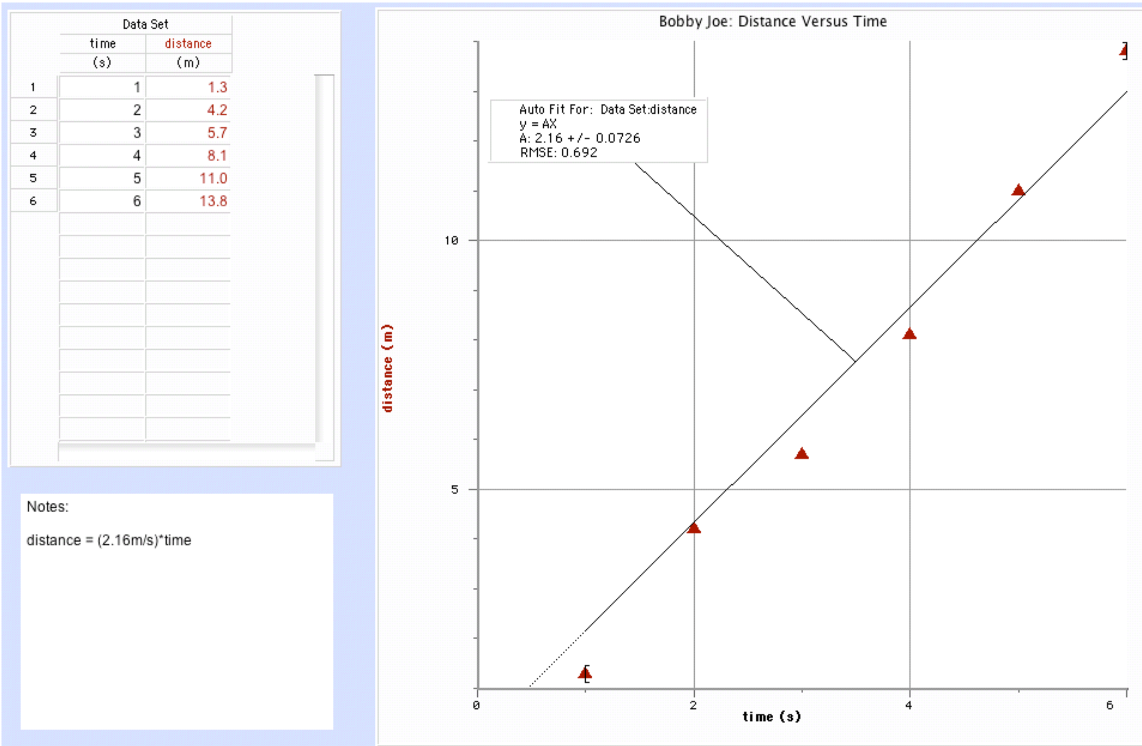
1. Point protectors missing.
2. Connecting data points with lines.
3. Regression line shown with data point line.
4. Multiple (0, 0) data points in attempt to "force" regression line through origin.
5. No column labels on data or graph.
6. No units on data or graph.
7. No label on graph.
8. Label does not include student's name.

9. Provides algebraic interpretation of data, not physical (shown here in Notes box)
10. Regression formula missing units on constants.

Other concerns:

11. Failure to linearize data to get a straight-line relationship unless otherwise called for.
12. Printing graph without data table on same page.
13. Failure to print use a landscape view.
14. Wasting paper and printer toner.

How to Prepare a Graph



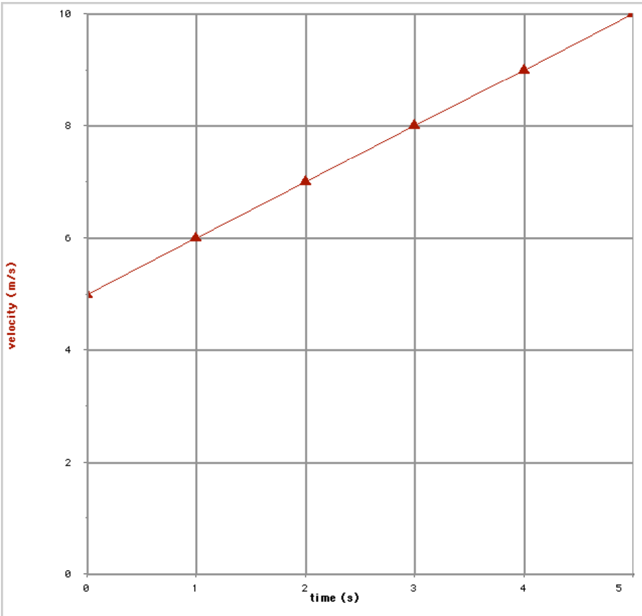
1. Point protectors clearly evident.
2. Not connecting data points with lines.
3. Regression line shown alone.
4. No (0, 0) data points; uses proportionality for regression line.
5. Column labels clearly evident on data and graph.
6. Units clearly evident on data or graph.
7. Label on graph.
8. Label includes student's name.
9. Provides physical interpretation of data, not algebraic (show here in Notes section).
10. Regression formula includes units on constants.

Other pointers:

11. Linearize data to get a straight-line relationship unless otherwise called for.
12. Print graph with data table on same page.
13. Print using a landscape view, avoid using portrait view.
14. Wasting paper and printer toner.

Interpreting Slopes, Areas, and Intercepts of Graphs

Sometimes slopes, areas, and intercepts in graphs have physical interpretations. At other times they have no real meaning at all. Consider how one can interpret meaningful terms in the following graph.



Here we have a linear relationship between velocity and time. The slope of the graph can be found from the relationship

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Because this is a linear relationship, arbitrarily choosing two pairs of (x, y) coordinates on the line and performing the calculation results in a slope of $1 m/s^2$. The slope represents the change in velocity with time which is nothing other than acceleration. Note that the calculated slope units are those of acceleration.

What does the area under the curve represent, say, from time equals $1s$ to time equals $5s$? Consider adding up a series of very narrow vertical columns to find total area. The height of each vertical column is velocity, v, at a particular time and the width of each column is Δt. The product of these terms is $v\Delta t$ which equals a distance. Hence the sum total of these rectangles giving the area under the curve during the specified time interval is the distance that the object has traveled from time $1s$ to $5s$. Can you see how the distance the object travels between $t = 1s$ and $t = 5s$ is $32m$? You can readily see this by adding the area of the rectangle below and to the right of the point (1, 6) (area = height x width = $(6m/s)(4s) = 24m$) to the triangle to the upper right of this point (area = 0.5 height x base = $0.5(4m/s)(4s) = 8m$). Note the $24m + 8m = 32m$.

The y-intercept can also be readily interpreted in the above graph. It is merely the value of the y term at time = $0s$. That is, the y-intercept is $5m/s$, the velocity of the object at $t = 0s$.

Not all graphs are so readily interpreted. Some graph's slopes, areas, and intercepts might well be meaningless. Consider the following examples, not all of which are meaningful. Can you tell which is which?

What does the area under the curve of an acceleration-versus-time graph represent?

What does the slope of an acceleration-versus-time graph represent?

What does the y-intercept of an acceleration-versus-time graph represent?

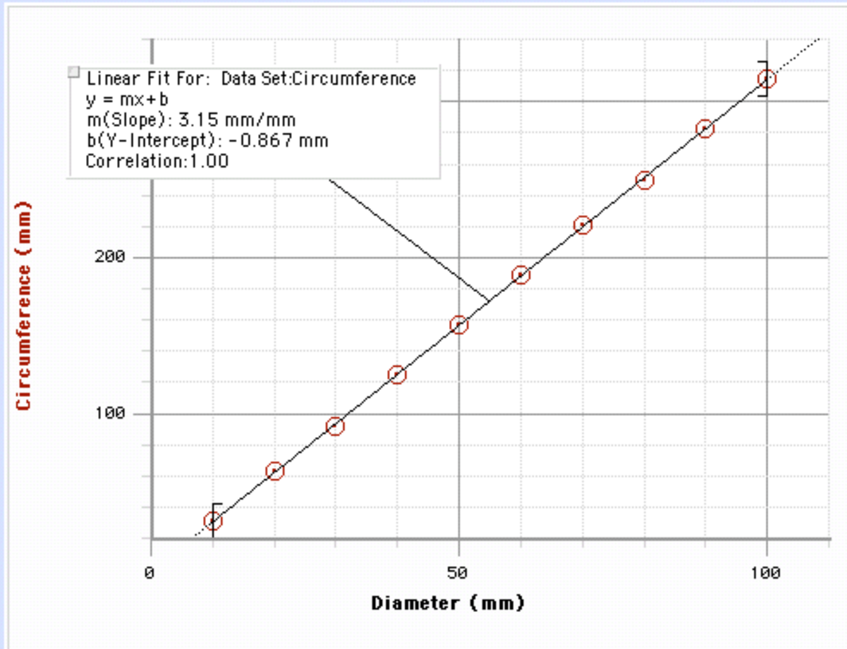
What does the area under the curve of a displacement-versus-time graph represent?

What does the slope of a displacement-versus-time graph represent?

What does the y-intercept of a displacement-versus-time graph represent?

Creating realistic models from data requires more than a blind "best fit" strategy. A physical model needs to be created from an algebraic model if the new model is to have any useful meaning. For instance, let's say that an experimenter measures the circumferences of a number of circular disks, as well as the diameters of these disks. There will be measurement error associated with each determination of circumference and diameter. A graph of circumference versus diameter produces the following result with the use of *Graphical Analysis*:

Data Set Name / Units	Diameter (mm)	Circum (mm)
1	10	31
2	20	63
3	30	92
4	40	125
5	50	157
6	60	189
7	70	221
8	80	249
9	90	283
10	100	315
11		



Note that an algebraic relationship results from a linear fit (y = mx + b) that is of the following form:

$$y = 3.15x - 0.867mm$$

Identifying *y* with circumference, *C*, and *x* with the diameter, *D*, the equation translates to the following form:

$$C = 3.15 \times D - 0.867mm$$

This relationship suggests that if diameter equals zero, the circumference would have some negative value. This clearly is incorrect in our physical world. If *D* = 0, then *C* must equal 0. The way to resolve is problem is to create a physical model. Clearly, the relationship is linear, but the regression line must pass through the origin (0,0).

A physical model can be created by forcing the regression line to pass through the origin. This is NOT done by including the data point 0,0; it IS done by using a different regression model, a proportionality (y = Ax), for the curve fit. Such a curve fit produces a different equation that we would call a physical model.

$$C = 3.14D$$

or more commonly

$$C = \pi D = 2\pi r$$

where *r* is the radius of the circle. This physical model is not weighed down by the non-zero y intercept. When interpreting graphical results in lab, be certain to use an interpretation that corresponds to the physical world as we know it.

Hypothetico-deductive approaches to scientific experimentation

Scientific Values

Scientists and engineers generally adhere to an informal set of values that serve as the basis of ethical conduct in the disciplines. Budding scientists and engineers are also expected to adhere to these rules of conduct. Unless these rules of conduct are expressly stated, students in an introductory lab course might never know them. What follows are one science educator author's interpretation of expected personal behaviors.

Cooperative attributes (and how students can demonstrate them in lab):

- *Be well informed* – Come to lab well prepared. This means not only to complete the required PreLab, but to have reviewed carefully the purpose of the indicated lab and given some thought to the required inquiry practices to be utilized in achieving stated goals. Read pertinent sections of the *Student Laboratory Handbook.*
- *Stay focused* – When in the laboratory setting, the emphasis should be on the first part of the word laboratory (labor) and not on the second (oratory). Work diligently on the task at hand, and see it through to completion with dispatch. Everyone should contribute to the best of his or her ability.
- *Evaluate alternatives* – Listen to others in your group; perhaps someone with an alternative approach might, in fact, be pointing out the best approach. Consider all arguments carefully before making a decision.
- *Take supportable positions* – Positions in lab should be based on evidence and critical thinking. If you hold a position contrary to that of you colleagues, justify it.
- *Being sensitive to others' positions* – Not everyone will agree in lab. If you disagree with another person in your group, then find a solution using reason and evidence. Criticize the idea, not the person.
- *Seek precision* – Don't be satisfied with "good enough." Try your best to minimize experimental error and improve your results. Strive for accuracy in word and action.
- *Proceed in a logical and orderly manner* – For many of you, this will be the first time you have been asked to design and conduct experiments. Think the process through carefully before collecting data.

Individual attributes (and how students can demonstrate them in lab):

- *Be well informed* – Come to lab prepared with an understanding of what it is you intend to accomplish and ideas for how you will accomplish it. This generally means completing the PreLab work, and reading through the lab guidelines in their entirety while attempting to understand. Also, don't forget to read the articles in the *Student Laboratory Handbook;* it has many good articles dealing with practical lab matters.
- *Stay focused* – While working in lab, it is important to stay focused on the task at hand. Avoid wayward discussions. The focus in *labor*atory should be on *labor* and not on oratory.
- *Be willing to evaluate alternatives* – While you will arrive in lab with your own ideas about how a task should be accomplished, carefully weigh ideas and comments by your lab partner(s) dealing with alternative approaches before coming to a firm decision.
- *Maintain only supportable positions* – You should always be sensitive to the positions of other. If, however, there is a justifiable reason for taking one approach over another, then stand your ground. Supportable positions are tenable. Ask for outside commentary from your lab instructor if you have reached an intractable disagreement about how to proceed.
- *Seek precision* – Always strive to be as accurate as would reasonably be expected of a scientist. There is no excuse for sloppy lab work. It would never be accepted by the scientific community in professional research, and won't be accepted in the introductory lab setting as well.
- *Proceed in a logical and orderly manner* – Plan your work before executing it. Failure to do so is like using a calculator to "solve a problem." Calculators don't solve problems; people do. The calculator literally calculates a solution, and can only do so after a human has reasoned out the approach to a solution. Merely using scientific hardware and software won't solve problems; nothing can replace good intellectual thought.

- *Be sensitive to others*– When working with your colleague(s), be considerate of his or her needs and concerns. Show proper respect; listen to others; think before you act.
- *Think critically* – There are many definitions of critical thinking, and any of the following would apply: “reasonable reflective thinking that is focused on deciding what to do and what to believe” OR “interpreting, analyzing or evaluating information, arguments or experiences with a set of reflective attitudes, skills, and abilities to guide our thoughts, beliefs and actions” OR “examining the thinking of others to improve our own.” Strive to apply critical thinking to your tasks.
- *Be intellectually honest* – Evaluate all pertinent evidence carefully and systematically. Avoid prejudicial decision-making.
- *Avoid unwarranted closure* – Avoid leaping to conclusions if data don’t support a particular position.
- *Demonstrate personal integrity* – Never “fudge” data to make it fit the expected outcomes. Never eliminate data unless there is a clear and defensible reason.